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Dislocation kink chain and multiphase periodic processes in the bounded sine-Gordon system

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Abstract. A finite chain of dislocation kinks in a crystal is considered as a string in the Peierls potential. Its behaviour is discussed in terms of multiphase periodic processes in the sine-Gordon system under adequate boundary conditions. It has been stated that in a particular case (dislocation segment containing kinks of the same width) the oscillations of the kink chain may be described by the Jacobi elliptic functions, whereas in a more general case (dislocation segment containing kinks of different width) the oscillations of the kink chain should be described by the Riemann theta functions. The presented description is thus a generalisation of that reported by Kovalev for the dynamics of a single Frenkel-Kontorova dislocation in a one-dimensional crystal with fixed boundaries. A soliton limit of the kink chain behaviour is also briefly discussed.

1. Introduction

The vibration of a dislocation segment in a crystal at low enough temperatures (when the thermal kink pairs are not created) can be described in a good approximation by the string model (Koehler 1952, Granato and Lücke 1956) provided that both ends of the segment lie in the same valley of the Peierls potential. In general, however, a segment does not lie in a single Peierls valley, but rather forms a dynamic system of specific kink configuration as shown in figure 1 for a static case, where all the kinks are of the same width. Then the behaviour of the dislocation segment is better described by the kink-chain model (Brailsford 1961, Seeger and Schiller 1962, Alefeld 1965). The analytical formula for the shape of a kinked dislocation segment is well known only in the simplest cases, i.e. for a one-kinked infinite dislocation line with a static kink (e.g. Hirth and Lothe 1972, Schoeck 1980) as well as with a moving one (e.g. Kosevich 1978, 1979, Pawełek 1985), but to our knowledge the general case of the many-kinked dislocation segment has not been considered in the literature.

In order to consider that case we start from the known fact, first pointed out by Seeger and Schiller (1966), Seeger and Engelke (1968) and recently rediscovered by Kosevich (1978, 1979), that the problem of the motion of a dislocation as a string in the Peierls potential without damping and external forces is equivalent to the problem of the dislocation motion in the atomic one-dimensional Frenkel-Kontorova (FK) model (Frenkel and Kontorova 1938, Frenkel 1972). This equivalence says that the atom motion in an infinite one-dimensional crystal in the FK model and the motion

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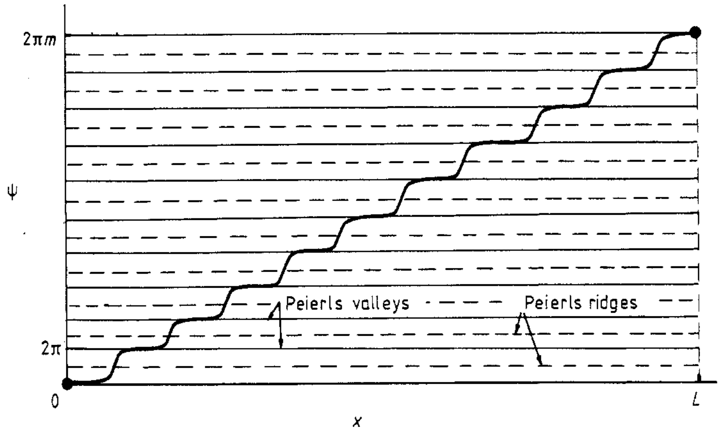

Figure 1. A static many-kinked dislocation segment spanned across the relief of the sinusoidal Peierls potential as the sine-Gordon system with boundary conditions $\psi(x=0)=0$ and $\psi(x=L)=2\pi m$, where ψ is the total displacement of atoms in dimensionless units.

of the infinite one-kinked dislocation line considered as a string in the Peierls potential are both governed by the sine-Gordon (SG) equation

$$\psi_{tt} - \psi_{xx} + \sin\psi = 0. \tag{1}$$

In (1) $\psi(x, t)$ is the function describing the dimensionless displacement of atoms (in the FK model) or the function describing the shape of an infinite one-kinked dislocation line (in the string model with Peierls potential and under boundary conditions $\psi(-\infty, t)=0$ and $\psi(+\infty, t)=a$, where a is the distance between adjacent Peierls valleys). Moreover, a simple physical interpretation of this equivalence has recently been given (e.g. Pawełek 1985), and also applied to the description of the thermodynamic equilibrium of kinks on a dislocation segment (Pawełek 1987a, b, 1988) as well as to a possible soliton description of the acoustic emission induced by plastic deformation of crystals (Pawełek 1987a, b, 1988). Thus each geometrical kink in a static chain (figure 1) as well as in a dynamic chain (where, in general, the kinks moving at different velocities are of different width) may be considered as a one-dimensional FK dislocation. Therefore a many-kinked dislocation segment, being a finite chain of kinks, can be treated as a string spanned across the Peierls potential relief under boundary conditions given by

$$\psi(x=0, t)=0 \qquad \text{and} \qquad \psi(x=L, t)=2\pi m \tag{2}$$

where m is the number of kinks in the chain.

2. Application of the Riemann theta function

In order to describe the dynamics of the many-kinked dislocation segment we should search for the multiperiodic solution of (1) which satisfies conditions (2). It appears that what is required for this purpose is just the Riemann theta function (e.g., Dubrovin *et al* 1976, Dubrovin and Natanson 1982, Kozel and Kotlarov 1976, Date and Tanaka 1976, Matveev 1976) as frequently discussed (Zagrodziński 1981, 1983, 1984a, b, Jaworski and Zagrodziński 1982) in the context of non-linear partial differential equations (NLPDE). The method of the Riemann θ function allows us to find a broader class of NLPDE solutions than those expressible in terms of elliptic Jacobi functions. The solutions are in general quasi-periodic which means that they are a 'non-linear' superposition of g-periodic processes, having, however, incommensurate periods as a rule. According to this method a general g-phase quasiperiodic solution of (1), obtained from the theory of Abelian integrals (Kozel and Kotlarov 1976), is given by

$$\psi(x, t) = 2\mathrm{i} \ln \frac{\theta(z+e/2 \mid B)}{\theta(z \mid B)} \tag{3}$$

where $\theta(z \mid B)$ is the Riemann theta function defined by

$$\theta(z \mid B) = \sum_{n \in Z^g} \exp(2\pi \mathrm{i}\langle n, z\rangle + \mathrm{i}\pi\langle n, Bn\rangle). \tag{4}$$

In (3) and (4) the vector z with the components

$$z_j = \kappa_j x + \lambda_j t + z_{0j} \qquad j = 1, \ldots, g \tag{5}$$

belongs to a g-dimensional complex vector space C^g, while the vectors n with integer components form the g-dimensional lattice Z^g. The matrix $B \in C^{g \times g}$ is the Riemann matrix and e is the g-dimensional vector, all components of which are equal to unity. The expression (3) is a solution of (1) if κ_j, λ_j, z_{0j} and B are determined by the Abelian integrals on the suitable Riemann surface, or according to another approach when an adequate system of dispersion relations is satisfied. The dispersion equations derived for the SG equation by Zagrodziński (1983) or in an equivalent form by Dubrovin and Natanson (1982) have the following form (for all $\mu \in Z_2^g$ where Z_2^g is the g-dimensional unit cube):

$$2 \sum_{p,q} f^{\mu}_{,pq}(0) A_{pq} - cf^{\mu}(0) = -(-1)^{\langle \mu, e\rangle} f^{\mu}(0) \tag{6}$$

where $f_{,pq} \equiv \partial^2 f/\partial p\, \partial q$, $A_{pq} = \kappa_p \kappa_q - \lambda_p \lambda_q$ and

$$f^{\mu}(w) = \theta(2w + B\mu \mid 2B) \exp(2\pi \mathrm{i}\langle \mu, w\rangle). \tag{7}$$

In general, for arbitrary complex z and B, expression (3) is also complex. Physically we are interested in real solutions only, and the reality condition imposes constraints on the admissible form of z and B (see table 1; quantities z and B are not unique since they can also be determined modulo symplectic transformation (Zagrodziński

Table 1. Conditions imposed upon B and z for the reality of ψ (τ_1 and $\tau_2 = \pm 1$ independently, β and η are real).

g	Type of wave	B	z
1	oscillating (O)	$\tau_1/2 + \mathrm{i}\beta$	$\mathrm{i}\eta$
	rotational (R)	$\mathrm{i}\beta$	$\mathrm{i}\eta + \tau_1/4$
2	oscillating-oscillating (OO)	$\begin{bmatrix} \tau_1/2 & 0 \\ 0 & \tau_2/2 \end{bmatrix} + \mathrm{i}\begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta_1 \\ \mathrm{i}\eta_2 \end{bmatrix}$
2	oscillating-rotational (OR)	$\begin{bmatrix} \tau_1/2 & 0 \\ 0 & 0 \end{bmatrix} + \mathrm{i}\begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta_1 \\ \mathrm{i}\eta_2 + \tau_2/4 \end{bmatrix}$
	rotational-rotational (RR)	$\mathrm{i}\begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta_1 + \tau_1/4 \\ \mathrm{i}\eta_2 + \tau_2/4 \end{bmatrix}$

1984b)). For $g=1$ there are two types of solutions, which by analogy with the motion of a pendulum can be named as oscillatory and rotational. In the case of $g=2$ we have three types of solutions which are a 'non-linear' superposition of the two elementary oscillating and/or rotational processes. For $g>2$ the situation is similar and there always exists a set of $g+1$ different types of real solutions. It should be noted that the oscillatory type of solution is bounded by 0 and 2π (or $\pm\pi$) which corresponds to the dislocation segment oscillating in a single Peierls valley. On the other hand, the rotational solution is not bounded, i.e. both ends of the dislocation segment lie in different Peierls valleys. It is clear that the boundary condition (2) can be satisfied by the rotational solution; thus in this paper we shall confine our attention to this type of solution.

3. Dynamics of a many-kinked dislocation segment

In order to describe the oscillations of many-kinked dislocations by the g-phase quasiperiodic solution expressed by the Riemann theta functions it is necessary to verify whether the real solution in form (3) fulfils both boundary conditions (2). Expression (3) is real provided $\theta(z+e/2\,|\,B)=\theta^*(z\,|\,B)$, where the asterisk denotes the complex conjugate, and then

$$\psi(x,t)=2\mathrm{i}\ln\frac{\theta^*(x,t\,|\,B)}{\theta(x,t\,|\,B)}=4\tan^{-1}\left(\frac{\operatorname{Im}\theta}{\operatorname{Re}\theta}\right). \tag{8}$$

Consequently the boundary condition (2) may be rewritten in the form

$$\operatorname{Im}\theta(0,t\,|\,B)=0 \qquad \text{for } -\infty<t<+\infty \tag{9}$$

and

$$\left.\begin{array}{ll}\operatorname{Re}\theta(L,t\,|\,B)=0 & m \text{ odd}\\ \operatorname{Im}\theta(L,t\,|\,B)=0 & m \text{ even}\end{array}\right\} \text{for } -\infty<t<+\infty. \tag{10}$$

A quasiperiodic solution fulfils the condition

$$\psi(x+L,t+T)=\psi(x,t) \qquad (\mathrm{mod}\ 2\pi) \tag{11}$$

where L and T are the space and time periods, respectively. This means that there exist vectors p, $r\in Z^g$, i.e. with integer components, such that by (5)

$$\kappa L+\lambda T=Bp+r \tag{12}$$

where $\kappa=(\kappa_1,\ldots,\kappa_g)$, $\lambda=(\lambda_1,\ldots,\lambda_g)$. In fact, due to the general formula (e.g., Zagrodziński 1982) for p, $r\in Z^g$

$$\theta(z+Bp+r\,|\,B)=\exp[-\mathrm{i}\pi(2\langle z,p\rangle+\langle p,Bp\rangle)]\theta(z\,|\,B) \tag{13}$$

we have just (11) since

$$\begin{aligned}2\mathrm{i}\ln&\frac{\theta[\kappa(x+L)+\lambda(t+T)+z_0+e/2\,|\,B]}{\theta[\kappa(x+L)+\lambda(t+T)+z_0\,|\,B]}\\ &=2\mathrm{i}\ln\frac{\theta(z+Bp+r+e/2\,|\,B)}{\theta(z+Bp+r\,|\,B)}\\ &=2\mathrm{i}\ln\frac{\theta(x+t+z_0+e/2\,|\,B)}{\theta(x+t+z_0\,|\,B)}+2\pi\langle e,p\rangle.\end{aligned} \tag{14}$$

Now, boundary condition (10) taken for $t \to t+T$ becomes

$$\theta(z+Bp+r\,|\,B) \pm \theta(z+Bp+r+e/2\,|\,B)$$
$$= \exp[-\mathrm{i}\pi(2\langle z, p\rangle + \langle p, Bp\rangle)][\theta(z\,|\,B) \pm \theta(z+e/2\,|\,B)(-1)^{\langle e,p\rangle}] \tag{15}$$

and will be satisfied by (9) if for $m =$ even (odd) $\langle e, p\rangle$ is simultaneously even (odd). Thus we have proved that having the solution satisfying boundary condition (9), i.e. at point $x = 0$, then condition (10) is also satisfied; however, we do not know whether m ought to be even or odd. On the other hand if the solution is pure periodic in space, then the question of the boundary conditions is trivial, but there arises the problem how to select among quasiperiodic solutions those which are pure periodic. In contrast to (11), the pure periodic (in space) solutions have the property

$$\psi(x+L, t) = \psi(x, t) \qquad (\mathrm{mod}\, 2\pi) \tag{16}$$

i.e. implying that $T = 0$ in equation (12). Since the SG equation is invariant under the Lorentz transformation, solution (8) will be periodic in space if there exists $\xi \in R$ such that for some $p, r \in Z^g$ the following relation holds

$$(\kappa \cosh \xi + \lambda \sinh \xi)L = Bp + r. \tag{17}$$

Since κ and λ are determined by the matrix B via dispersion equations (6), relation (17) represents an additional restriction upon the class of admissible B matrices. It is clear that, if the periodic (in space) solution satisfies $\psi(0, t) = 0$, it satisfies also condition $\psi(L, t) = 2\pi m$ provided that $\langle e, p\rangle = m$. Thus it is seen from the vectorial equation (17) that the reduction of the quasiperiodic process to the periodic one poses the question of an adequate choice of ξ that should meet the following conditions

$$\tanh \xi = \frac{q_i\kappa_k - q_k\kappa_i}{q_k\lambda_i - q_i\lambda_k} \qquad i > k = 1, \ldots, g \tag{18}$$

where q_k are the components of vector $q = Bp + r$, $p, r \in Z^g$. We shall prove that for given $p, r \in Z^g$, solution (8) can be reduced by the Lorentz transformation to the solution either purely periodic in space or purely periodic in time. Indeed, there exist such L and $T \in R$ for the solution (8) that for given $p, r \in Z^g$, equation (12) is satisfied. Let us now assume that $|L| > |T|$ and denote $K^2 = L^2 - T^2$. Thus

$$L = K \cosh \xi \qquad T = K \sinh \xi \qquad \tanh \xi = T/L \tag{19}$$

and (12) becomes

$$(\kappa \cosh \xi + \lambda \sinh \xi)K = Bp + r. \tag{20}$$

This just means that (17) holds and the solution (8) after the Lorentz transformation with parameter ξ is purely periodic in space. If $|L| < |T|$ holds, then a similar procedure will give a solution purely periodic in time. In terms of spectral representation related to the SG equation (Zagrodziński 1984, Dubrovin 1981), the Lorentz transformation with parameter ξ means that all the points of the main spectrum $(E_1, \ldots, E_{2g})$ are transformed into new ones $(E'_1, \ldots, E'_{2g})$ and

$$E'_i = E_i \frac{1+\tanh \xi}{1-\tanh \xi} \qquad i = 1, \ldots, 2g. \tag{21}$$

This statement allows us to exclude the case $|L| = |T|$, since then all the points of the main spectrum E_i would be reduced to single points: zero or infinity, which is impossible. Thus, in general, solution (8) satisfying a boundary condition at point $x = 0$, i.e. condition (9), satisfies also the condition at $x = L$, i.e. condition (10), although the 'parity' of m is unknown. Moreover there is the possibility of the reduction of a quasiperiodic solution to the pure periodic one by a proper choice of the Lorentz parameter ξ. (We have no proof that the considered solution will be periodic in space but the experience of the authors gained so far indicates that we have always two families of solutions—one reducible to periodic in space and the other in time.)

Thus the determination of the wavevectors κ_i and angular frequencies λ_i in the solution (12) leads to a general description of the dynamics of a finite many-kinked dislocation segment. In order to find the effective solutions we should solve in each particular case the system of dispersion equations (6) with respect to κ_j, λ_j, $j = 1, 2, 3$, for a given B matrix. A more simplified way to find the solutions (8) can be realised by the assumption of the approximated form of the dispersion relations (e.g., Zagrodziński and Jaworski 1982), similar to the case of our other recent calculations (Pawełek and Jaworski 1988). However, in this paper we intend to present rather schematic illustrations of these solutions and the detailed calculations will not be quoted here. On the other hand the general solutions, being a dynamic shape of a many-kinked dislocation segment, can be more easily deduced from the simplest case of the oscillations of a one-kinked dislocation and its interaction with fixed boundaries. Hence this case will be discussed below in more detail and some other more general examples deduced from it will be considered in a schematic way only.

4. Examples

We consider firstly the oscillations of the one-kinked ($m = 1$) dislocation segment under boundary conditions (2). In this case the solution (8) is of a two-periodic character and thus, according to general relations between Riemann theta function and Jacobi elliptic functions (derived recently, e.g., by Zagrodziński 1982), it can be expressed in terms of Jacobi elliptic functions. It follows from these relations that the oscillations of the one-kinked dislocation are described just by the same formula as found by Kovalev (1979) for a single FK dislocation oscillating in a one-dimensional crystal with fixed boundaries

$$\psi(x, t) = 4 \tan^{-1}[(k'/l')^{1/2} \operatorname{dn}(\alpha t, 1) \operatorname{tn}(\gamma x, k)]. \tag{22}$$

In the above equation

$$\alpha = [(1 - k'/l')(1 - k'l')]^{-1/2} \qquad \gamma = \alpha(k'/l')^{1/2} \qquad L = K(k)/\alpha \tag{23}$$

whereas $\operatorname{dn}(u, k)$, $\operatorname{tn}(u, k) \equiv \operatorname{sn}(u, k)/\operatorname{cn}(u, k)$ are the Jacobi elliptic functions of the argument u and modulus k ($k' = (1 - k^2)^{1/2}$, $l' = (1 - l^2)^{1/2}$, $0 < k, l < 1$), and $K(k)$ is the complete elliptic integral of the first kind. The solution (22) was discussed also by Costabile *et al* (1978) and DeLeonardis *et al* (1980) in relation to the problem of boundary conditions imposed on the sine-Gordon equation. They considered a narrow class of SG equation solutions expressible by the elliptic functions and commonly known as the Lamb ansatz (Lamb 1971). Expression (22) describes the two-phase periodic solution since it represents two identical waves travelling in opposite directions with equal velocities (figure 2).

From this behaviour we can deduce now the behaviour of a many-kinked dislocation segment satisfying the boundary conditions (2).

We consider below the following simple examples:

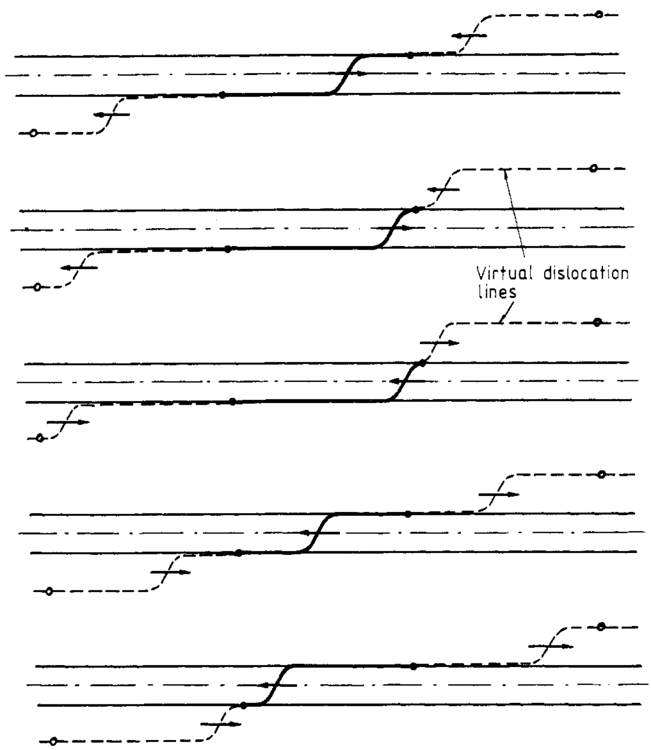

Figure 2. Schematic illustration of the oscillations of the one-kinked dislocation segment.

(i) a one-periodic static kink chain (see figure 1),

(ii) an even number of kink-trains travelling pairwise in opposite directions with equal velocities, and

(iii) a 'non-linear' superposition of cases (i) and (ii).

Note that each kink-train may have more than one kink within the dislocation segment. Thus an m-kinked segment corresponds, in general, to a g-periodic solution, where $1 \leqslant g \leqslant 2m$. For instance, figure 3 shows a many-kinked ($m=6$) dislocation segment consisting of two kink-trains travelling in opposite directions with the same velocities. Thus, when choosing $g=2$, the case $m=6$ admits the solution in terms of the Lamb ansatz, similarly to the case illustrated in figure 2. However, according to our knowledge, it is impossible to express the general multiperiodic solution in terms of the Jacobi elliptic functions, as in the case of the Lamb ansatz, since the technique of separability of the variables does fail.

Below we consider the simplest non-trivial case which cannot be solved by means of the Lamb ansatz and Jacobi elliptic functions. Let us take a two-kinked dislocation segment ($m=2$) having one kink stationary and the other moving with velocity v. Similarly to figure 2, the moving kink is accompanied by a virtual kink travelling in the opposite direction at the same velocity. Thus the total number of phases is $g=3$. The solution belongs to the rotational type (see table 1), hence $z_{0j}=\frac{1}{4}$, while κ_j, λ_j and B_{ij} are purely imaginary. In addition, the symmetry of the problem implies: $\lambda_1=0$ (stationary process), $\lambda_2=-\lambda_3$, $B_{12}=B_{13}$, $B_{22}=B_{33}$ (equal velocities in opposite directions). Figure 4 shows schematically oscillations of the dislocation segment consisting

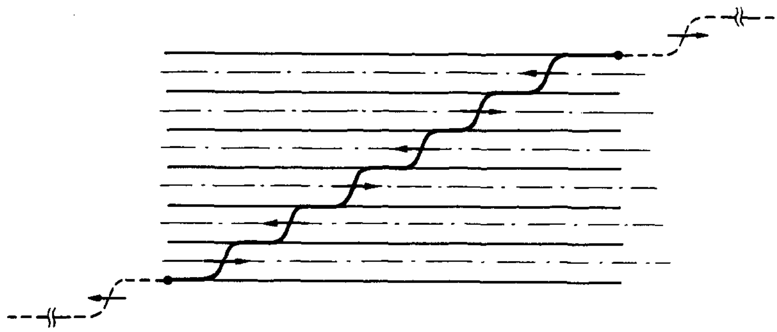

Figure 3. An example of the many-kinked dislocation segment ($m = 6$) as two kink-trains ($g = 2$) travelling in opposite directions. Observe that the absolute values of all velocities are equal.

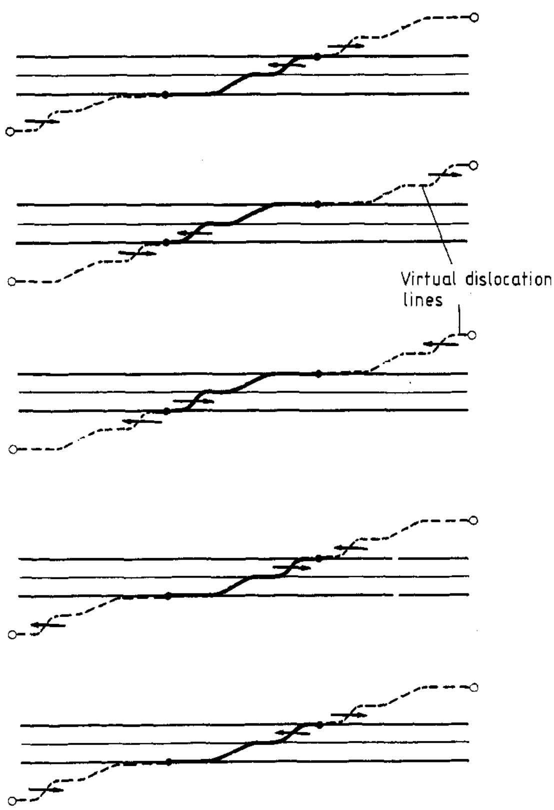

Figure 4. Schematic illustration of the oscillations of a two-kinked dislocation segment consisting of one static kink and one moving kink ($m = 2$, $g = 3$).

of one moving and one stationary kink. The stationary kink is at rest when no interaction takes place, but it experiences a phase shift during the interaction with the moving kink (marked by an arrow). In a similar way one can also find other solutions for many-kinked dislocation segments being a non-linear superposition of the cases (i) and (ii). Furthermore we may consider a slightly general case, illustrated in figure 5, where both kinks are moving but at different velocities (still $m = 2$). One can see two pairs of kink-trains where both kink-trains in each pair are travelling in opposite directions at the same velocities but different for each pair (thus $g = 4$). The above examples show clearly that the solutions (3) expressed by the Riemann theta function form a much broader class than those obtained by the Lamb ansatz and expressed by the Jacobi elliptic functions. We believe that these examples reflect quite well the dynamic behaviour of at least those kink chains which are long and sloping to the direction of Peierls valleys at not so great angles, i.e. when the kink chain may be approximately regarded as a superposition of the isolated kinks. For great slope angles the non-linear superposition is important and the dynamic shape of the kink chain should be calculated in detail.

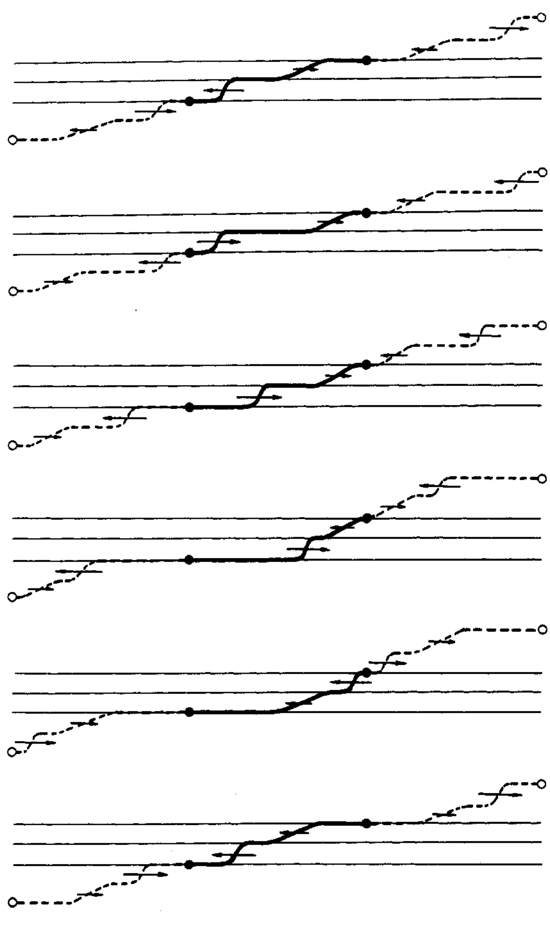

Figure 5. Schematic illustration of the oscillations of a two-kinked dislocation segment consisting of both moving kinks at different velocities ($m = 2$, $g = 4$); note that in a general case the velocities may be incommensurate, resulting in quasiperiodic oscillations in time.

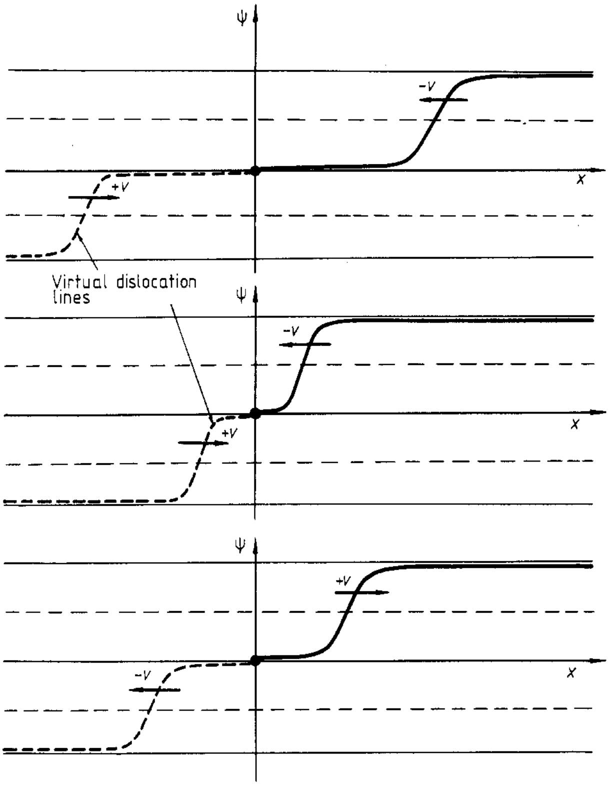

Figure 6. Schematic illustration of the motion of the half-infinite (i.e. fixed at the point $x = 0$) one-kinked dislocation line. The interaction with a boundary is described by the soliton–soliton collision as in the unbounded sine-Gordon system.

Moreover, one can also discuss briefly the soliton limit of the above quasiperiodic solutions in the spirit of the limiting procedure introduced by Zagrodziński and Jaworski (1982) or Zagrodziński (1984a) by applying the so-called T-function. These limiting situations would then have a physical interpretation as a many-kinked infinite dislocation line. In a particular case when all the kinks have different velocities, the g-periodic solution will tend to the g-soliton process along the infinite dislocation line. For instance, in the limiting case of two-periodic solution (22), i.e. when $L \to \infty$ ($k, l \to 1$, $k'/l' \to v$), it is reduced to the well known pure soliton–soliton solution (Kovalev 1979, DeLeonardis *et al* 1980):

$$\psi(x, t) = 4 \tan^{-1}\left(\frac{v \sinh[x/(1-v^2)^{1/2}]}{\cosh[vt/(l-v^2)^{1/2}]}\right) \tag{24}$$

which describes the motion and interaction of the half-infinite one-kinked dislocation line with a fixed boundary at $x = 0$ ($\psi(0, t) = 0$ and $\psi(+\infty, t) = 2\pi$; see figure 6).

5. Summary and conclusions

In our concluding remarks we would like to emphasise that the description of the dynamics of an m-kinked dislocation segment presented here is valid also for the dynamics of many FK dislocations in a one-dimensional crystal with fixed boundaries. Thus it is a generalisation of that presented by Kovalev (1979) for a single FK dislocation only. Moreover, we would like to point out that such an approach to the dislocation kink chain, as presented here, implies that there is no necessity to distinguish the string model (in the sense of a dislocation segment spanned across the relief of the Peierls potential) from the kink-chain model since in the latter the behaviour of the chain is a specific form of the behaviour of dislocation string oscillating across many Peierls barriers under adequate boundary conditions. A better agreement of the kink-chain model with low-temperature experimental data (e.g., on internal friction below the Bordoni peak) than of the classical string model (i.e. in the sense of the dislocation segment vibrating in a single Peierls valley) is due to the fact that the prevailing contribution to this phenomenon is provided by many-kinked dislocation segments rather than by non-kinked or by the segments with a small number of kinks (Brailsford 1961, Alefeld 1965). At high temperatures, above the Bordoni peak, the predictions of the kink-chain and string models give similar results (Seeger and Schiller 1962, Suzuki and Elbaum 1964, Seeger and Engelke 1968) since the kink chain behaves strictly as a classical string because the Peierls barriers may be disregarded at this temperature.

Eventually, the following conclusions may be formulated.

(i) A many-kinked dislocation segment, being a finite chain of kinks, can be treated as a string oscillating across the relief of the sinusoidal Peierls potential.

(ii) The two-periodic solution (22) expressed by the Jacobi elliptic functions may describe the oscillations of the m-kinked dislocation segment in particular cases only when all kinks are of the same width; then the admissible kink velocities are $\pm v$.

(iii) The multiphase solution (8), expressed by the Riemann theta function, may describe the dynamics of the m-kinked dislocation segment in a more general case; the order of the θ-function cannot exceed $2m$.

(iv) In the limiting cases the multiperiodic solutions can be reduced to the multisoliton solutions and thus they describe the dynamics of the infinite many-kinked dislocation line.

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